1-page summary of the HH model:

The Hodgkin-Huxley model of a neuron consists of a system of four coupled first-order differential equations. The four dependent variables are (V, n, m, h); these are, in order, the membrane potential, a gating variable for the potassium channel, and two gating variables for the sodium channel. Set $V = 0$ outside the cell (though Hodgkin and Huxley adopted a different voltage convention) and the differential equations take the form:

$$C\,\frac{dV}{dt} = I_{\text{input}}(t) - \bar{g}_{\text{K}}n^4(V - V_{\text{K}}) - \bar{g}_{\text{Na}}m^3h(V - V_{\text{Na}}) - \bar{g}_{\text{L}}(V - V_{\text{L}}) \tag{1}$$

$$\frac{dn}{dt} = -\frac{n - n_\infty(V)}{\tau_n(V)} \tag{2}$$

$$\frac{dm}{dt} = -\frac{m - m_\infty(V)}{\tau_m(V)} \tag{3}$$

$$\frac{dh}{dt} = -\frac{h - h_\infty(V)}{\tau_h(V)}\,, \tag{4}$$

where the externally applied current $I_{\text{input}}(t)$ is a prescribed function. Typical values of the parameters are:

Nernst potentials:	$V_{\text{K}} = -77\,\text{mV}$,	$V_{\text{Na}} = +60\,\text{mV}$,	$V_{\text{L}} = -54.4\,\text{mV}$
maximum conductances:	$\bar{g}_{\text{K}} = 36\,\mu\text{mho}$,	$\bar{g}_{\text{Na}} = 120\,\mu\text{mho}$,	$\bar{g}_{\text{L}} = 0.3\,\mu\text{mho}$,

and $C = 1\,\text{nF}$ (based on a neuron with $0.1\,\text{mm}^2$ area). The nonlinear functions $\mu_\infty(V)$, $\tau_\mu(V)$ — where $\mu = n, m, h$ — are plotted in Figure 1, and are based on experimental measurements. Often, the differential equations (2)–(4) for the gating variables are written instead in the form:

$$\frac{d\mu}{dt} = \alpha_\mu(V)\big(1 - \mu\big) - \beta_\mu(V)\,\mu \qquad \text{where}\,\mu = n, m, h\,.$$

The V-dependent functions are related by:

$$\mu_\infty(V) = \frac{\alpha_\mu(V)}{\alpha_\mu(V) + \beta_\mu(V)}\,, \qquad \tau_\mu(V) = \frac{1}{\alpha_\mu(V) + \beta_\mu(V)} \qquad \text{for } \mu = n, m, h\,.$$

A typical choice of the $\alpha_\mu(V)$ and $\beta_\mu(V)$ functions, again based on fitting data, is:

$$\alpha_n(V) = \frac{0.1 - 0.01(V + 65)}{e^{1 - 0.1(V+65)} - 1} \qquad \alpha_m(V) = \frac{2.5 - 0.1(V + 65)}{e^{2.5 - 0.1(V+65)} - 1} \qquad \alpha_h(V) = 0.07e^{(-V-65)/20}$$

$$\beta_n(V) = 0.125e^{(-V-65)/80} \qquad \beta_m(V) = 4e^{(-V-65)/18} \qquad \beta_h(V) = \frac{1}{e^{3 - 0.1(V+65)} + 1}$$

where α_μ and β_μ are measured in ms^{-1}, and V in mV. Note that one must always be careful to use $\alpha_\mu(V)$ and $\beta_\mu(V)$ functions that are consistent with the voltage convention.

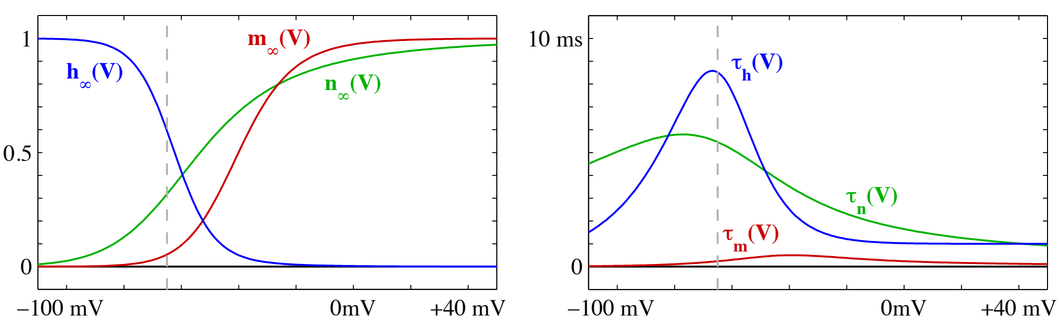

Figure 1: Plots of the functions $\mu_\infty(V)$ and τ_μ, where $\mu = n, m, h$.